

BEHAVIOUR AND ANALYSIS OF LATERALLY LOADED PILES SUBJECTED TO STATIC LATERAL LOADS

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ABSTRACT

Deep foundations are used when the soil is very weak below the ground surface. The most common types of deep foundations are piles, piers and cassettes. A pile subjected to lateral loading is one of the classes of problems that involve interaction of soil and structures.

The main aim of the project is to calculate structural parameters like Slope(θ), Deflection(Y), Moment (M), Shear(S), Soil reaction(P) etc., at each and every point of the pile. When the soil near the ground surface is not capable of supporting a structure, deep foundations are required to transfer the loads to deeper strata.

In this project we are using Finite Difference Method (F.D.M). This method is applicable for regular shapes. In this project we are taking ALUMINIUM and PVC as pile materials. Finally structural parameter values are compared with C program using matrix inversion method and also by using MS excel.

Key Words: Pile, FDM. C program,

1. INTRODUCTION

A pile subjected to lateral loading is one of a class of problems that involve the interaction of soils and structures. Soil-structure interaction is encountered in every problem in foundation engineering, but in some cases the structure is so stiff that a solution can be developed assuming nonlinear behavior for the soil and no change in shape for the structural unit. But for a pile under lateral loading, a solution cannot be obtained without accounting for the deformation of both the pile and the soil. The deflection of the pile and the lateral resistance of the soil are interdependent; therefore, because of the

Nonlinearity of the soil, and sometimes of the pile, iterative techniques are almost always necessary to achieve a solution for a particular case of loading on the pile.

In Finite Difference Method at each of the pivotal point of intervals, an equation expressing the differential equation by the finite differences can be established. A set of simultaneous equations are developed and they are solved using the boundary conditions at the pivotal points. The computations can be easily handled mechanically on calculators or digital computers.

1.1 Finite Difference Method:

Finite Difference Method can be advantageously employed for complex loading, boundary configurations, mathematical expressions, load distribution sectional properties etc...Partial differential equations are invariably required in solving problems of two – dimensional structural elements can be made easier by using FDM. This method is used for determination of moments, shear forces, deflections, buckling of columns which involve the differential equations.

FDM is applicable for statically determinate and statically indeterminate structures, vibration problems, torsional problems and beams on elastic foundations.

1.2 Objective and Scope

Engineers understood early that a pile under lateral load would act as a beam. Hetenyi (1946) wrote a book giving the solution of the differential equation for a beam on a foundation, with a linear relationship between pile deflection and soil response. In the early 1950s, Shell Oil Company was planning to install an offshore platform at Block 42 in 25m (82ft) of water near the Louisiana coast, where the soil was

predominantly soft clay. A procedure was available for computing lateral forces on the platform during a hurricane.

The scope of present project work is to evaluate deflections, shear, moments, and soil interaction of the laterally loaded pile using finite difference method. This can be done by dividing the pile into number of nodal points. Using Finite Difference equations and substituting the appropriate boundary conditions, we can calculate unknown deflections, Moment, Shear and soil reaction. Selection of too many incremental nodal points may lead to erroneous solutions, because difference between successful increments disappears. However, use of computer program with understanding leads to good solution of practical problems.

2. LITERATURE REVIEW

FORMULATION OF THE EQUATION BY FINITE DIFFERENCES OF LATERALLY LOADED PILE:

The derivation for the differential equation for the beam column on a foundation was given by Hetenyi (1946).

$$\frac{dM}{dx} + P_x \frac{dy}{dx} - V_v = 0$$

And making the indicated substitutions we get generalized governing differential equation for a pile under lateral loading which is as follows.

$$E_p I_p \frac{d^4 y}{dx^4} + P_x \frac{d^2 y}{dx^2} + p = 0$$

The derivatives of differential equation needed for formulation of finite difference equations are as follows.

y – Deflection;

$$\frac{dy}{dx} - \text{slope}(s)$$

$$EI \frac{d^2 y}{dx^2} = \text{moment}(M)$$

$$EI \frac{d^3 y}{dx^3} = \text{shear}(V)$$

$$EI \frac{d^4 y}{dx^4} = \text{soil reaction}(P)$$

The derivatives can be written in difference form as follows.

$$\frac{dy}{dx} = \frac{y_{i-1} - y_{i+1}}{2h}$$

$$\frac{d^2 y}{dx^2} = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$$

$$\frac{d^3 y}{dx^3} = \frac{y_{i-2} - 2y_{i-1} + 2y_{i+1} - y_{i+2}}{2h^3}$$

$$\frac{d^4 y}{dx^4} = \frac{y_{i-2} - 4y_{i-1} + 6y_i - 4y_{i+1} + y_{i+2}}{h^4}$$

The governing differential equation for a pile under lateral loading is as follows.

$$EI \frac{d^4 y}{dx^4} - p + w = 0$$

Where P – Soil reaction (P = KY_n)

W = distributed load along length of pile

There is no additionally acting external distributed loaded on laterally loaded pile. Only self weight of pile is considered.

$$y_{i-2} - 4y_{i-1} + 6y_i - 4y_{i+1} + y_{i+2} = \left(\frac{Ky_i h^4}{EI} - \frac{wh^4}{EI} \right)$$

Boundary conditions:

Top of the pile

Moment $\Rightarrow EI \frac{d^2 y}{dx^2} = P_t \times e$

Shear $\Rightarrow EI \frac{d^3 y}{dx^3} = P_t$

Bottom of the pile

Moment is zero at pile tip.

$$EI \frac{d^2 y}{dx^2} = 0$$

3. METHODOLOGY:

The program is validated for a hypothetical problem as given below. Consider a free headed flexible which is subjected to lateral load of 57.01N, 129.3 N, 191.08 N, 269.57 N , 401.27 N with eccentricity of 0.25m above ground line. The pile is having design parameters

Table 3.1 Design parameters

PILE MATERIAL	PVC	ALUMINIUM
FLEXTURAL RIGIDITY (E_pI_p)NMM²	3.990*10 ⁷	1.136*10 ⁹
SOIL MODULUS (K) Mpa	0.34	0.34
RELATIVE STIFFNESS (K_R)	1.789*10 ⁻⁴	5.093*10 ⁻³

Calculate the deflections of nodal points, assuming 4 sub intervals. Consider a free headed flexible pile which is divided in to 4 parts with

nodal points 0, 1, 2, 3, 4 respectively which is shown in figure as follows.

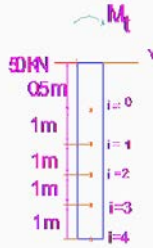


Fig 3.1 Representation of nodal points

4. RESULTS AND DISCUSSIONS:

4.1. DEFLECTION CALCULATIONS (Y):

For aluminium:

Sub grade modulus $K = 0.34 \text{ Mpa}$
 $h = 250\text{mm}; EI = 1.136 \times 10^9 \text{ N/m}^2$
 Eccentricity (e) = 35 mm

a) For load $P_t = 57.01 \text{ N}$

Substituting the above parameters in equations we get the set of equations for free headed flexible pile which is as follows.

$$\begin{aligned} 0.831y_0 - 4y_1 + 2y_2 &= -1.3486 \\ -2y_0 + 3.831y_1 - 4y_2 + y_3 &= -0.1097 \\ y_0 - 4y_1 + 4.831y_2 - 4y_3 + y_4 &= 0 \\ y_1 - 4y_2 + 3.831y_3 - 2y_4 &= 0 \\ 2y_2 - 4y_3 + 0.831y_4 &= 0 \end{aligned}$$

b) For load $P_t = 129.3 \text{ N}$

Substituting the above parameters in equations we get the set of equations for free headed flexible pile which is as follows.

$$\begin{aligned} 0.831y_0 - 4y_1 + 2y_2 &= -3.05884 \\ -2y_0 + 3.831y_1 - 4y_2 + y_3 &= -0.24898 \\ y_0 - 4y_1 + 4.831y_2 - 4y_3 + y_4 &= 0 \\ y_1 - 4y_2 + 3.831y_3 - 2y_4 &= 0 \\ 2y_2 - 4y_3 + 0.831y_4 &= 0 \end{aligned}$$

Similarly the calculations were done for the PVC material, corresponding loading conditions and results are obtained as discussed.

4.2. SLOPE CALCULATIONS (S):

The general formula for slope

$$\frac{dy}{dx} = \frac{y_{i-1} - y_{i+1}}{2h}$$

4.2.1 For aluminium:

a) For load 57.01 N

$$\frac{P_t e h^2}{EI} = 0.1097$$

Substitute y_0, y_1, y_2, y_3, y_4 in the above equations we get

$$S_0 = -0.022445, S_1 = -0.0161019, S_2 = 9.87478 \times 10^{-3}$$

$$S_3 = 0.0176094, S_4 = 0.0237048$$

b) For load 129.3 N

$$\frac{P_t e h^2}{EI} = 0.24898$$

Substitute y_0, y_1, y_2, y_3, y_4 in the above equations we get

$$S_0 = -0.050908, S_1 = -0.036521, S_2 = 2.239866 \times 10^{-3}$$

$$S_3 = 0.039939, S_4 = 0.053763192$$

Similarly the calculations were done for the PVC material, corresponding loading conditions and results are obtained as discussed.

4.3. MOMENT CALCULATIONS (M) :

We know general equation for moment

$$EI \frac{d^2y}{dx^2} = EI \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$$

4.3.1 For aluminium:

a) For load 57.01 N

Substitute y_0, y_1, y_2, y_3, y_4 in the above equations we get

$$M_0 = 1.9954 \text{ N-M}, M_1 = 11.1190 \text{ N-M}, M_2 = 0.4190 \text{ N-M}, M_3 = -0.0661 \text{ N-M}, M_4 = 0 \text{ N-M}$$

b) For load 129.3 N

Substitute y_0, y_1, y_2, y_3, y_4 in the above equations we get

$$M_0 = 4.5255 \text{ N-M}, M_1 = 25.218 \text{ N-M}, M_2 = 0.9504 \text{ N-M}, M_3 = -0.1498 \text{ N-M}, M_4 = 0 \text{ N-M}$$

Similarly the calculations were done for the PVC material, corresponding loading conditions and results are obtained as discussed.

4.4 SHEAR CALCULATIONS (V):

General equation for shear

$$EI \frac{d^3y}{dx^3} = EI \frac{y_{i-2} - 2y_{i-1} + 2y_{i+1} - y_i + 2}{2h^3}$$

4.4.1 For aluminium:

a) For load 57.01 N

Substitute y_0, y_1, y_2, y_3, y_4 in the above equations we get

$$V_0 = 57.01 \text{ N}, V_1 = 54.3669 \text{ N}, V_2 = 55.7288 \text{ N}, V_3 = 55.8073 \text{ N}, V_4 = 55.790 \text{ N}$$

b) For load 129.3 N

Substitute y_0, y_1, y_2, y_3, y_4 in the above equations we get

$$V_0 = 129.3 \text{ N}, V_1 = 123.305 \text{ N}, V_2 = 126.39 \text{ N}, V_3 = 126.561 \text{ N}, V_4 = 126.541 \text{ N}$$

Similarly the calculations were done for the PVC material, corresponding loading conditions and results are obtained as discussed.

4.5 SOIL REACTION CALCULATIONS (P):

General equation for soil reaction

$$EI \frac{d^4y}{dx^4} = \frac{y_{i-2} - 4y_{i-1} + 6y_i - 4y_{i+1} + y_{i+2}}{h^4}$$

4.6.1 For aluminium:

a) For load 57.01 N

Substitute y_0, y_1, y_2, y_3, y_4 in the above equations we get

$P_0 = 0 \text{ N/M}, P_1 = 2.6431 \text{ N/M}, P_2 = -1.367 \text{ N/M}$

$P_3 = -0.0737 \text{ N/M}, P_4 = 0.0088 \text{ N/M}$

b) For load 129.3 N

Substitute y_0, y_1, y_2, y_3, y_4 in the above equations we get

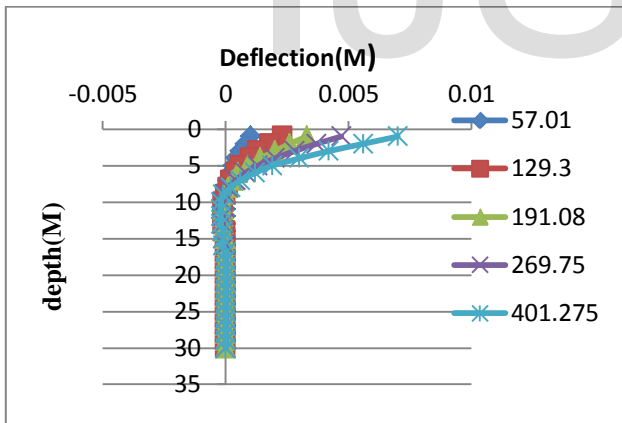
$P_0 = 0 \text{ N/M}, P_1 = 5.9947 \text{ N/M}, P_2 = -3.0890 \text{ N/M}$

$P_3 = -0.1667 \text{ N/M}, P_4 = 0.0703 \text{ N/M}$

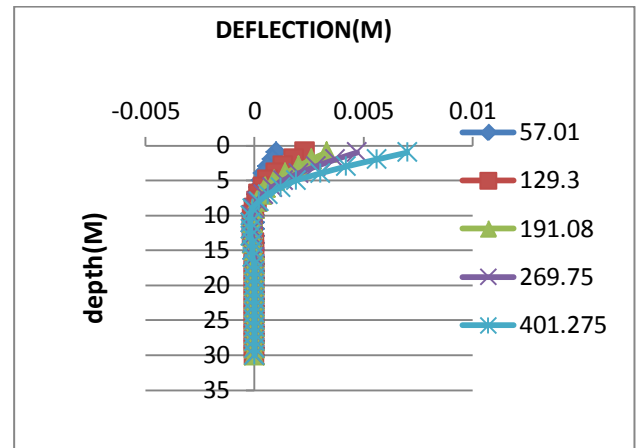
Similarly the calculations were done for the PVC material, corresponding loading conditions and results are obtained as discussed.

VALIDATION OF RESULTS:

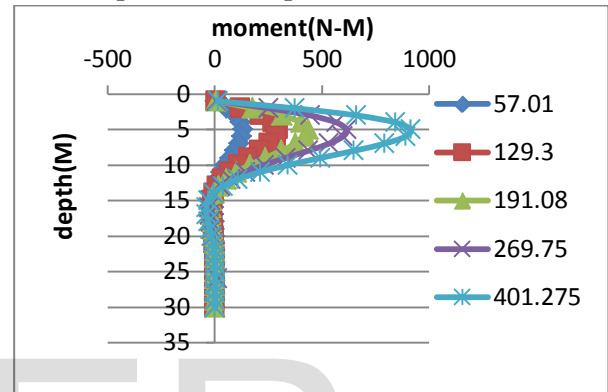
The results are compared with C program for the materials Aluminium and PVC and discussed in the graphs



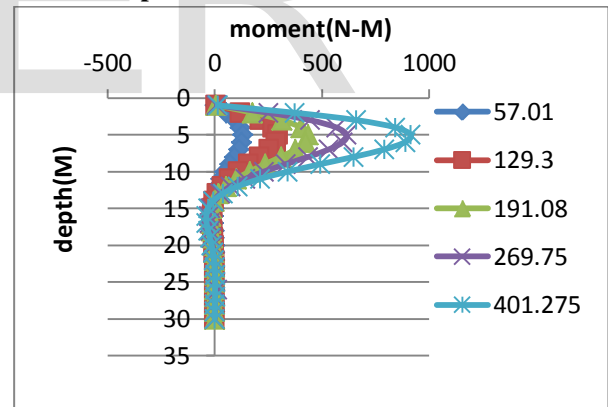
4.5.1 Graph between Depth and Deflection for aluminium pile



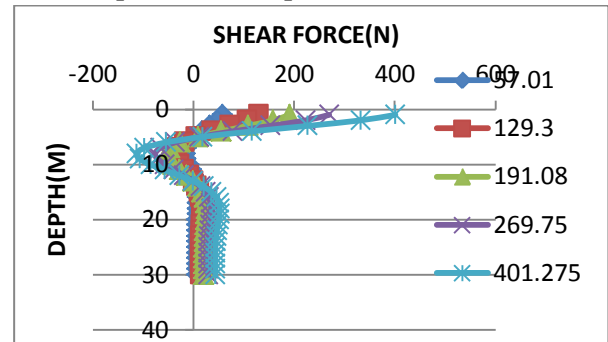
4.5.2 Graph between Depth and deflection for PVC



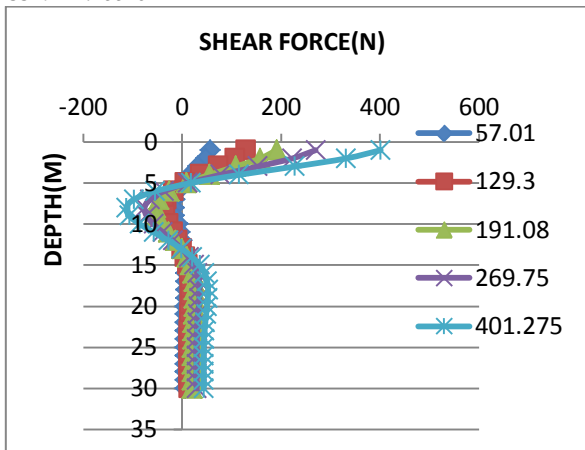
4.5.3 Graph between Depth and Moment for aluminium pile



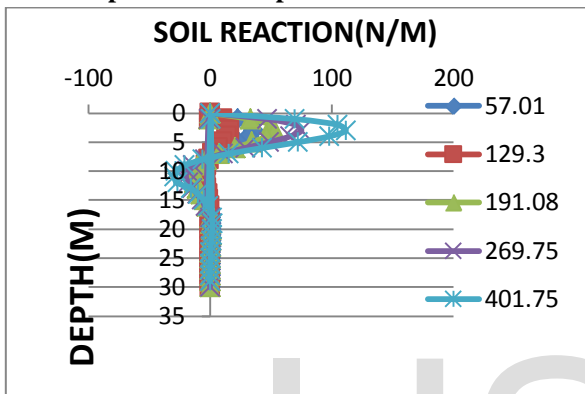
4.5.4 Graph between Depth and Moment for PVC



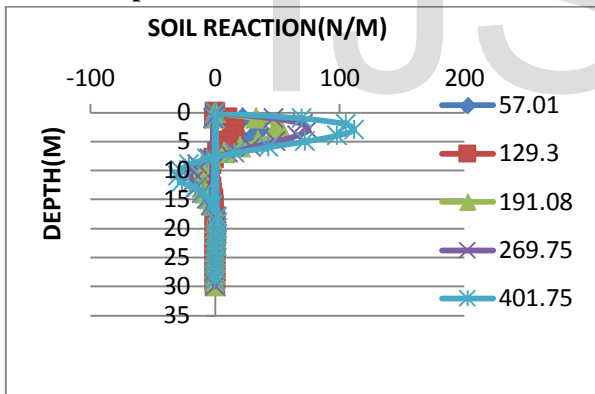
4.5.5 Graph between Depth and Shear for aluminium pile



4.5.6 Graph between Depth and Shear for PVC



4.5.7 Graph between Depth and Soil reaction for aluminium pile



4.5.8 Graph between Depth and Soil reaction for PVC

5. CONCLUSIONS:

For a pile under lateral loading, a solution cannot be obtained without accounting deformation of both pile and soil. The deflection of the pile and the lateral resistance are interdependent. Because of non linearity of soil, sometimes of the pile interactive techniques are always necessary to achieve a solution for a particular case of loading on pile. In this project a computer code has been developed for the

analysis of laterally loaded pile using finite difference method. First pile is subdivided into m increments in which $(m+1)$ equations can be written leading to $(m+5)$ unknowns. Substituting appropriate boundary conditions fictitious nodes at top and bottom portion of pile has been eliminated leading to $(m+1)$ unknowns and $(m+1)$.

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